N87-17827

On Incorporating Damping and Gravity Effects in Models...

by

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On Incorporating Damping and Gravity Effects in Models of Structural Dynamics of the SCOLE Configuration

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Presented at
The 3rd Annual SCOLE* Workshop
NASA Langley Research Center
Hampton, Virginia
November 17, 1986

*Spacecraft Control Laboratory Experiment(SCOLE)

On Incorporating Damping and Gravity Effects in Models of Structural Dynamics of the SCOLE Configuration

ABSTRACT

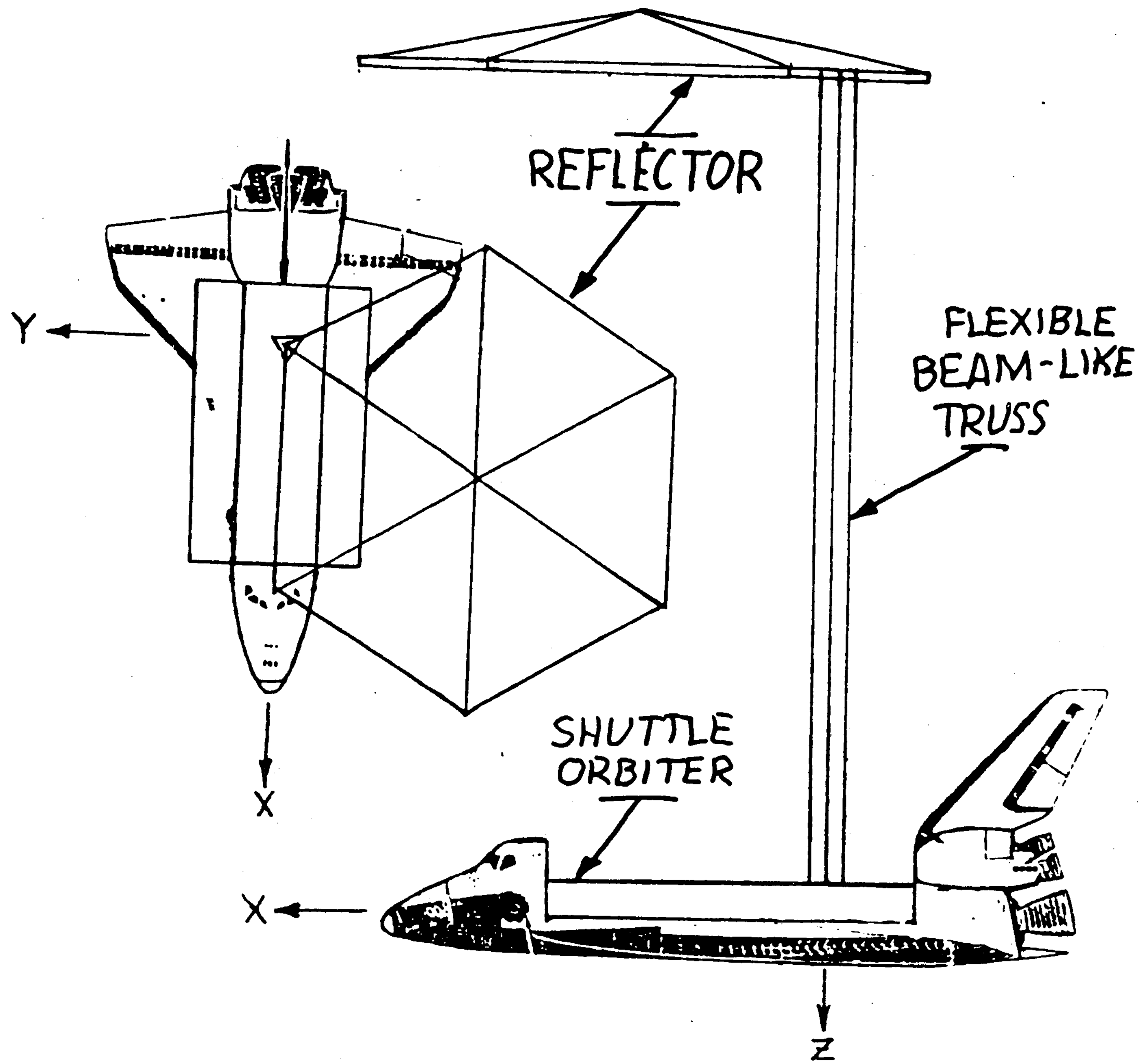
The damping for structural dynamics models of flexible spacecraft is usually ignored and then added after modal frequencies and mode shapes are calculated. It is common practice to assume the same damping ratio for all modes, although it is known that damping due to bending and that due to torsion are different. Mass effects on damping are sometimes ignored.

It is the purpose of this paper to examine two ways of including damping in the modeling process from its onset. First, the partial derivative equations of motion are analyzed for a pinned-pinned beam with damping. The end conditions are altered to handle bodies with mass and inertia for the SCOLE configuration. Second, a massless beam approximation is used for the modes with low frequencies, and a clamped-clamped system is used to approximate the modes for arbitrarily high frequency. The model is then modified to include gravity effects and is compared with experimental results.

OUTLINE

- Introductory Remarks
- SCOLE Configuration
- Partial Differential Equations
- Pinned-Pinned System with Damping
- Free-Free System with End Bodies & Damping
- Massless Beam Approximation
- Gravity Effects
- Comparison of Model Frequencies
- Concluding Remarks

CONFIGURATION



Equations of Motion

Shuttle (and Reflector) Body

$$\dot{\omega}_1 = -I_1^{-1}(\tilde{\omega}_1 I_1 \omega_1 - M_1 - M_{1,Beam})$$

$$\dot{v}_1 = (F_1 + F_{1,Beam})/m_1$$

$$\dot{T}_1^T = -\tilde{\omega}_1 T_1^T$$

Roll (and Pitch) Beam Bending

$$\rho A_\phi \frac{d^2 u_\phi}{dt^2} - C I_\phi \frac{d^3 u_\phi}{ds^2 dt} + E I_\phi \frac{d^4 u_\phi}{ds^4} = \sum_{n=1}^{4} [f_{\phi,n}\,\delta(s-s_n) + g_{\phi,n}\frac{d\delta}{ds}(s-s_n)]$$

Yaw Beam Torsion

$$\rho I_\psi \frac{d^2 u_\psi}{dt^2} + C I_\psi \frac{d^3 u_\psi}{ds^2 dt} - G I_\psi \frac{d^2 u_\psi}{ds^2} = \sum_{n=1}^{4} g_{\psi,n}\,\delta(s-s_n)$$

Beam Elongation

$$\rho A \frac{d^2 u_z}{dt^2} + C_z A \frac{d^2 u_z}{ds\,dt} - E A \frac{d^2 u_z}{ds^2} = \sum_{n=1}^{4} f_{z,n}\,\delta(s-s_n)$$

Damping Considerations

- The Classical Damping, $\frac{du^5}{ds^4 dt}$ Yields Excessive

  Excessive Damping at Higher Mode Numbers

- The Term, $\frac{du^3}{ds^2 dt}$ is Consistent with experimental

  Data.

- The Practice of Post-Analysis Addition of

  Damping Ignores Effects of Mass, Stress Type.

- Damping Must be Included from the Start.

Distributed Parameter Model of SCOLE with "Proportional Damping"

- Start with Pinned-Pinned Beam with Damping
- Add Bodies with Inertia at Ends
- Model Acceleration of Frame as Inertial Loading
- Extend in Three Dimensions to SCOLE Configuration.
- Yields Infinite-Order, Modal, State Equations.

Distributed Parameter System

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{21}^{*} & A_{22}^{*} \end{bmatrix}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix}\begin{bmatrix} M_1 \\ M_4 \\ F_1 \\ F_4 \end{bmatrix}$$

$$A_{21}^{*} = \{B_M\begin{bmatrix} I_1 & 0 \\ 0 & I_4 \end{bmatrix}R\}^{-1}[A + B\begin{bmatrix} -pA & 0 \\ \frac{pA}{L} & \frac{pA}{L} \end{bmatrix}Q]$$

$$A_{22}^{*} = \{B_M\begin{bmatrix} I_1 & 0 \\ 0 & I_4 \end{bmatrix}R\}^{-1}$$

$$B_{22}^{*} = \{B_M\begin{bmatrix} I_1 & 0 \\ 0 & I_4 \end{bmatrix}R\}^{-1}[-B_M \,\vdots\, B_w\begin{bmatrix} -pA & 0 \\ \frac{pA}{L} & \frac{pA}{L} \end{bmatrix} + B_A]$$

$$A = \begin{bmatrix} 0 & 1 & & & & & \\ -\varpi_1 & -2\delta\varpi_1 & & & & & \\ & & 0 & 1 & & & \\ & & -\varpi_2 & -2\delta\varpi_2 & & & \\ & & & & 0 & 1 & \\ & & & & -\varpi_3 & -2\delta\varpi_3 & \\ & & & & & & \ddots \end{bmatrix}$$

$$B_M = \begin{bmatrix} 0 & 0 \\ \frac{2\pi^2}{\rho A L^3} & -\frac{2\pi^2}{\rho A L^3} \\ 0 & 0 \\ \frac{8\pi^2}{\rho A L^3} & \frac{8\pi^2}{\rho A L^3} \\ 0 & 0 \\ \frac{18\pi^2}{\rho A L^3} & -\frac{18\pi^2}{\rho A L^3} \\ \vdots & \vdots \end{bmatrix} \quad B_W = \begin{bmatrix} 0 & 0 \\ \frac{-8}{\rho A L} & \frac{-4}{\rho A} \\ 0 & 0 \\ 0 & \frac{4}{\rho A} \\ 0 & 0 \\ \frac{-8}{\rho A L} & \frac{-4}{\rho A} \\ \vdots & \vdots \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & \cdots \end{bmatrix}$$

$$B_A = \left[-B_M \;\middle|\; B_W \begin{bmatrix} \frac{\rho A}{m_1} & 0 \\ \frac{\rho A}{L m_1} & \frac{\rho A}{L m_4} \end{bmatrix} \right]$$

$$Q = \begin{bmatrix} 0 & 0 \\ -\frac{EI\pi^2}{L^2} & \frac{EI\pi^2}{L^2} \\ 0 & 0 \\ -\frac{4EI\pi^2}{L^2} & -\frac{4EI\pi^2}{L^2} \\ 0 & 0 \\ \vdots & \vdots \end{bmatrix}$$

Massless Beam Model

- Exact Static Deflection
- Approximates Low-Frequency Modes
- Nonlinear Kinematics
- Linearized State Space, Modal Model
- Classical Damping(Working Proportional)
- Extended to n-Body Network

Gravity Effects

- Assume Cubic Deflection of Beam
- Express Potential Energy due to the Raising of End Body
- Relatc to Stiffness Matrices of the Massless Beam Model
- Incorporate Gravity Effects in the Stiffness Matrices
- Gravity Effects Larger than Structural Stiffness

Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{A} = \begin{bmatrix} 0 & a_{12} & 0 & a_{14} & 0 & a_{16} & 0 & a_{18} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & 0 & a_{34} & 0 & a_{36} & 0 & a_{38} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 0 & a_{56} & 0 & a_{58} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & a_{72} & 0 & a_{74} & 0 & a_{76} & 0 & a_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ 0 & 0 & 0 & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} \\ 0 & 0 & 0 & 0 \\ b_{71} & b_{72} & b_{73} & b_{74} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x=\begin{bmatrix} \mathbf{w}_1 \\ \mathbf{E}_1 \\ \mathbf{V}_1 \\ \mathbf{R}_1 \\ \mathbf{w}_4 \\ \mathbf{E}_4 \\ \mathbf{V}_4 \\ \mathbf{R}_4 \end{bmatrix} \qquad u=\begin{bmatrix} \mathbf{F}_1 \\ \mathbf{m}_1 \\ \mathbf{F}_4 \\ \mathbf{m}_4 \end{bmatrix}$$

$$a_{12} = I_1^{-1}[-M_u\tilde{r}_1 + M_\angle + \tilde{r}_1 F_\angle - \tilde{r}_1 F_u \tilde{r}_1] = -a_{16}$$

$$a_{14} = I_1^{-1}[M_u + \tilde{r}_1 F_u] = -a_{18}$$

$$a_{32} = \frac{1}{m_1}[-F_u\tilde{r}_1 + F_\angle] = -a_{36}$$

$$a_{34} = \frac{1}{m_1}[F_u] = -a_{38}$$

$$a_{52} = I_4^{-1}[-M_u\tilde{r}_4 + M_\angle + \tilde{r}_4 F_\angle - \tilde{r}_4 F_u \tilde{r}_4] = -a_{56}$$

$$a_{54} = I_4^{-1}[M_u + \tilde{r}_4 F_u] = -a_{58}$$

$$a_{72} = \frac{1}{m_4}[-F_u\tilde{r}_4 + F_\angle] = -a_{76}$$

$$a_{74} = \frac{1}{m_4}[F_u] = -a_{78}$$

I - Moment of Inertia
m - Mass
r - Coordinates of attach point
$\tilde{r}$ - Cross product operator, $r\times$
$M_u, M_\angle, F_u, F_\angle$ - Stiffness Matrices
1 - Denotes the Shuttle body
4 - Denotes the reflector body
$u, \angle$ - Beam deflection and slope

Stiffness Matrices

$$F_u = \begin{bmatrix} -\frac{12EI}{L^3} - \frac{6W^*}{5L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} - \frac{6W^*}{5L} & 0 \\ 0 & 0 & -\frac{EA}{L} \end{bmatrix}$$

$$F_{\acute{u}} = \begin{bmatrix} 0 & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

Stiffness Matrices

$$M_{u'} = \begin{bmatrix} -\frac{4EI}{L} - \frac{2WL}{15}^{*} & 0 & 0 \\ 0 & -\frac{4EI}{L} - \frac{2WL}{15}^{*} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{bmatrix}$$

$$M_{u} = \begin{bmatrix} 0 & \frac{6EI}{L^2} + \frac{W}{10}^{*} & 0 \\ \frac{6EI}{L^2} + \frac{W}{10}^{*} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

```
      PROGRAM TWOBOD
      REAL I1,I4,I1IN,I4IN,M1,M4,MU,MANG,L,MASS1,MASS4
      DIMENSION I1(13),I1IN(13),I4(13),I4IN(13),A(580),RAT1(7),
     *RAT1T(13),RAT4(7),RAT4T(13),FU(13),FANG(13),MU(13),MANG(13),
     *DUM(580),DUN(580),EREAL(30),EIMAG(30),EVEC(580),DUO(580)
C...................................DEFINE INERTIA MATRICES................
      CALL SET(A,24,24)
      CALL SET(I1,3,3)
      I1(5)=905443.
      I1(7)=-145393.
      I1(9)=6789100.
      I1(11)=-145393.
      I1(13)=7086601.
      CALL SPIT(I1,3H I1)
      CALL MAKE(DUM,I1)
      CALL INVR(I1,I1IN)
      CALL MAKE(I1,DUM)
      CALL SET(I4,3,3)
      I4(5)=4969.
      I4(9)=4969.
      I4(13)=9938.
      CALL SPIT(I4,3H I4)
      CALL MAKE(DUM,I4)
      CALL INVR(I4,I4IN)
      CALL MAKE(I4,DUM)
C.........................................DEFINE ATTACH POINT VECTOR, MATRIX.
      CALL SET(RAT1,3,1)
      CALL TILDA(RAT1,RAT1T)
      CALL SET(RAT4,3,1)
      RAT4(5)=-18.75
      RAT4(6)=32.5
      CALL TILDA(RAT4,RAT4T)
      MASS4=12.42
C................................ADD HALF OF BEAM MASS TO REFLECTOR BODY...
      AD=MASS4/(MASS4+12.42*.5)
      CALL ADD(AD,RAT4T,-1.,RAT4T,DUM)
      CALL SPIT(DUM,4H DUM)
      CALL MULT(DUM,DUM,DUN)
      CALL SPIT(DUN,4H DUN)
      CALL ADD(1.,I4,-12.42,DUN,I4)
      AD=.5*12.42/(12.42+.5*12.42)
      CALL ADD(AD,RAT4T,-1.,RAT4T,DUM)
      CALL MULT(DUM,DUM,DUN)
      CALL ADD(1.,I4,-6.21,DUN,I4)
      CALL SPIT(I4,5H I4NU)
      CALL INVR(I4,I4IN)
      MASS1=6366.46+.09556*130./2.
      MASS4=MASS4+.09556*130/2.
C...............................BEAM SECTION CHARACTERISTICS...............
      EI=40000000.
      GJ=40000000.
      EA=100000000.
      L=130.
C...............................SET UP FORCE/DEFLECTION MATRIX..............
      CALL SET(FU,3,3)
      FU(5)=-12.*EI/(L*L*L)
      FU(9)=-12.*EI/(L*L*L)
      FU(13)=-EA/L
C...............................SET UP FORCE/SLOPE ANGLE MATRIX..............
      CALL SET(FANG,3,3)
      FANG(6)=6.*EI/(L*L)
      FANG(8)=FANG(6)
C...............................SET UP MOMENT/DEFLECTION MATRIX..............
      CALL MAKE(MU,FANG)
```

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C..........................SET UP MOMENT/SLOPE ANGLE MATRIX.................
      CALL SET(MANG,3,3)
      MANG(5)=-4.*EI/L
      MANG(9)=-4.*EI/L
      MANG(13)=-GJ/L
      CALL SPIT(FU,3H FU)
      CALL SPIT(FANG,5H FANG)
      CALL SPIT(MU,3H MU)
      CALL SPIT(MANG,5H MANG)
C........................CALCULATE ELEMENTS IN "A" MATRIX...................
      CALL MULT(RAT1T,FU,DUM)
      CALL MULT(DUM,RAT1T,DUN)
      CALL MULT(RAT1T,FANG,DUM)
      CALL ADD(1.,DUM,-1.,DUN,DUN)
      CALL ADD(1.,MANG,1.,DUN,DUN)
      CALL MULT(MU,RAT1T,DUM)
      CALL ADD(-1.,DUM,1.,DUN,DUN)
      CALL MULT(I1IN,DUN,DUM)
      CALL INSERT(1,4,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(1,16,DUM,A)
      CALL MULT(RAT1T,FU,DUM)
      CALL ADD(1.,MU,1.,DUM,DUN)
      CALL MULT(I1IN,DUN,DUM)
      CALL INSERT(1,10,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(1,22,DUM,A)
      CALL IDENT(DUM,3)
      CALL INSERT(4,1,DUM,A)
      CALL INSERT(10,7,DUM,A)
      CALL INSERT(16,13,DUM,A)
      CALL INSERT(22,19,DUM,A)
      CALL MULT(FU,RAT1T,DUM)
      CALL ADD(-1.,DUM,1.,FANG,DUN)
      AD=1./MASS1
      CALL ADD(AD,DUM,0.,DUM,DUM)
      CALL INSERT(7,4,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(7,16,DUM,A)
      CALL ADD(AD,FU,0.,FU,DUM)
      CALL INSERT(7,10,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(7,22,DUM,A)
C                         ................A52..........
      CALL MULT(RAT4T,FU,DUM)
      CALL MULT(DUM,RAT4T,DUN)
      CALL MULT(RAT4T,FANG,DUM)
      CALL ADD(1.,DUM,-1.,DUN,DUN)
      CALL ADD(1.,DUN,1.,MANG,DUN)
      CALL MULT(MU,RAT4T,DUM)
      CALL ADD(-1.,DUM,1.,DUN,DUN)
      CALL MULT(I4IN,DUN,DUM)
      CALL INSERT(13,16,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(13,4,DUM,A)
      CALL MULT(RAT4T,FU,DUN)
      CALL ADD(1.,DUN,1.,MU,DUN)
      CALL MULT(I4IN,DUN,DUM)
      CALL INSERT(13,22,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(13,10,DUM,A)
      CALL MULT(FU,RAT4T,DUM)
      CALL ADD(-1.,DUM,1.,FANG,DUN)
      AD=1./MASS4
      CALL ADD(AD,DUN,0.,DUN,DUM)
      CALL INSERT(19,16,DUM,A)
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```
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(19,4,DUM,A)
      CALL ADD(AD,FU,0.,FU,DUM)
      CALL INSERT(19,22,DUM,A)
      CALL ADD(-1.,DUM,0.,DUM,DUM)
      CALL INSERT(19,10,DUM,A)
C...............................CALCULATE EIGEN VALUES, MODE SHAPES.........
      CALL EIGEN(A,EREAL,EIMAG,EVEC,IERR)
      CALL SPIT(EREAL,5H REAL)
      CALL SPIT(EIMAG,5H IMAG)
  123 FORMAT(I10,E15.6)
C...............................PRINT NON-ZERO ELEMENTS OF "A" MATRIX...........
      DO 10 I=4,580
      IF(A(I)**2-.00000000001)11,11,12
   12 PRINT 123,I,A(I)
   11 CONTINUE
   10 CONTINUE
      END
--EOI/TOP--
??
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I1        1          2          3
  1  .9054E+06  0000E+00-.1454E+06
  2  .0000E+00 .6789E+07 .0000E+00
  3 -.1454E+06 .0000E+00 .7087E+07
.4341883E+20
I4        1          2          3
  1  .4969E+04 .0000E+00 .0000E+00
  2  .0000E+00 .4969E+04 .0000E+00
  3  .0000E+00 .0000E+00 .9938E+04
.2453788E+12
DUM       1          2          3
  1  .0000E+00 .0000E+00-.1083E+02
  2  .0000E+00 .0000E+00-.6250E+01
  3  .1083E+02 .6250E+01 .0000E+00
DUN       1          2          3
  1 -.1174E+03-.6771E+02 .0000E+00
  2 -.6771E+02-.3906E+02 .00[illegible]E+00
  3  .0000E+00 .0000E+00-.1564E+03
I4NU      1          2          3
  1  .9342E+04 .2523E+04 .0000E+00
  2  .2523E+04 .6424E+04 .0000E+00
  3  .0000E+00 .0000E+00 .1577E+05
.8458959E+12
FU        1          2          3
  1 -.2185E+03 .0000E+00 .0000E+00
  2  .0000E+00-.2185E+03 .0000E+00
  3  .0000E+00 .0000E+00-.7692E+06
FANG      1          2          3
  1  .0000E+00 .1420E+05 .0000E+00
  2  .1420E+05 .0000E+00 .0000E+00
  3  .0000E+00 .0000E+00 .0000E+00
MU        1          2          3
  1  .0000E+00 .1420E+05 .0000E+00
  2  .1420E+05 .0000E+00 .0000E+00
  3  .0000E+00 .0000E+00 .0000E+00
MANG      1          2          3
  1 -.1231E+07 .0000E+00 .0000E+00
  2  .0000E+00-.1231E+07 .0000E+00
  3  .0000E+00 .0000E+00-.3077E+06
PEAL      1
  1 -.1344E-04
  2 -.9690E-06
  3 -.2861E-08
  4 -.6211E-14
  5  .6211E-14
  6  .2861E-08
  7  .9690E-06
  8  .1344E-04
  9  .0000E+00
 10  .0000E+00
 11  .0000E+00
 12  .0000E+00
 13  .0000E+00
 14  .0000E+00
 15  .0000E+00
 16  .0000E+00
 17  .0000E+00
 18  .0000E+00
 19  .0000E+00
 20  .0000E+00
 21  .0000E+00
 22  .0000E+00
```

23	.0000E+00
24	.0000E+00
IMAG	1
1	.0000E+00
2	.0000E+00
3	.0000E+00
4	.0000E+00
5	.0000E+00
6	.0000E+00
7	.0000E+00
8	.0000E+00
9	.8682E-09
10	-.8682E-09
11	.1398E-05
12	-.1398E-05
13	.1621E+01
14	-.1621E+01
15	.2328E+01
16	-.2328E+01
17	.5821E+01
18	-.5821E+01
19	.1124E+02
20	-.1124E+02
21	.1617E+02
22	-.1617E+02
23	.3764E+03
24	-.3764E+03

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8	-.136379E+01
10	-.699510E-02
15	.157361E-01
20	.136379E+01
22	.699510E-02
27	-.157361E-01
33	-.181286E+00
38	.209176E-02
45	.181286E+00
50	-.209176E-02
56	-.279804E-01
58	-.435624E-01
63	.322851E-03
68	.279804E-01
70	.435624E-01
75	-.322851E-03
77	.100000E+01
102	.100000E+01
127	.100000E+01
158	-.342839E-01
170	.342839E-01
183	-.342839E-01
195	.342839E-01
208	-.120708E+03
220	.120708E+03
227	.100000E+01
252	.100000E+01
277	.100000E+01
296	.753974E+05
297	.433556E+05
298	.101820E+02
302	.667765E+00
303	-.170050E+01
304	.231538E+04
308	-.753974E+05
309	-.433556E+05
310	-.101820E+02
314	-.667765E+00
315	.170050E+01

316	-.231538E+04
320	.433556E+05
321	.252605E+05
322	.678424E+02
326	-.247271E+01
327	.667765E+00
328	.133580E+04
332	-.433556E+05
333	-.252605E+05
334	-.678424E+02
338	.247271E+01
339	-.667765E+00
340	-.133580E+04
344	.168886E+02
345	.292737E+02
346	.390243E+02
350	-.450364E+00
351	-.259825E+00
356	-.168886E+02
357	-.292737E+02
358	-.390243E+02
362	.450364E+00
363	.259825E+00
377	.100000E+01
402	.100000E+01
427	.100000E+01
441	-.762218E+03
442	-.381109E+03
446	.117264E+02
453	.762218E+03
454	.381109E+03
458	-.117264E+02
464	-.762218E+03
466	-.219870E+03
471	.117264E+02
476	.762218E+03
478	.219870E+03
483	-.117264E+02
488	.134182E+07
489	.774127E+06
496	.412868E+05
500	-.134182E+07
501	-.774127E+06
508	-.412868E+05
527	.100000E+01
552	.100000E+01
577	.100000E+01

REVERT NORMAL END.

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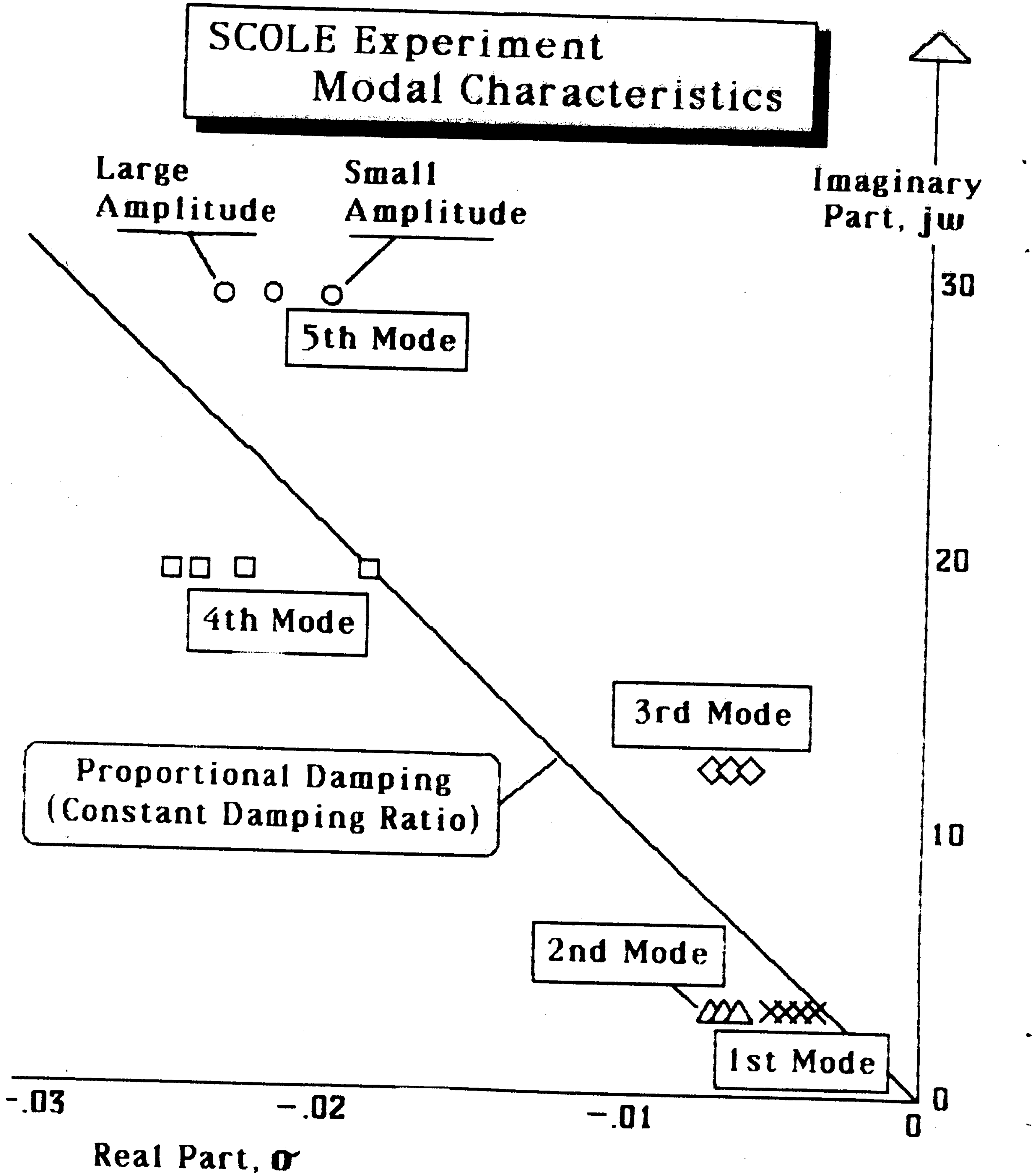
Static Deflection

Number of Modes	Error in Deflection
1	39%
2	24
3	17
4	13
5	11
6	9
7	8
8	7
9	6
:	:
67	1

Comparison of Modal Frequencies

Mode	P.D.E.	Finite El.	Lumped Mass *Clamped
1	.278	.276	.258
2	.314	.301	.370
3	.812	.810	.926
4	1.18	1.18	1.79
5	2.05	2.05	2.57
6	4.76	4.77	4.28*
7	5.51	5.52	4.28*
8	12.3	12.4	11.89*


SCOLE Experiment
Modal Characteristics
Large Amplitude
Small Amplitude
Imaginary Part, jw
30
5th Mode
20
4th Mode
3rd Mode
Proportional Damping (Constant Damping Ratio)
10
2nd Mode
1st Mode
0
-.03
-.02
-.01
0
Real Part, σ

Concluding Remarks

- An Infinite-Order State Space Model was Developed which Incorporates "Proportional" Damping.
- A Lumped Mass Model of SCOLE was Developed which Includes Gravity Effects and Classical Damping. Extended to n-Body Modeling.
- Modal Frequencies are Compared for the SCOLE using Different Methods.
- Items Remain to be Addressed Before SCOLE Modeling is Complete.

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